Low energy parameters of the $K\overline{K}$ and $\pi\pi$ scalar-isoscalar interactions

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Abstract

Threshold expansions of the $\pi\pi$ and $K\overline{K}$ spin 0 and isospin 0 scattering amplitudes are performed. Scattering lengths, effective ranges and so–called volume parameters are evaluated. Good agreement with the existing experimental data for the $\pi\pi$ scalar–isoscalar amplitude is found. An importance of future accurate measurements of the $K\overline{K}$ threshold parameters is stressed. New data are needed to understand the basic features of the scalar mesons.

Kaon–antikaon interactions are very poorly known. A characteristic feature of the $K\overline{K}$ interactions is a presence of the annihilation processes in which a creation of the $\pi\pi$ pairs plays a very important role. Thus the $K\overline{K}$ and $\pi\pi$ channels are coupled together and should be treated simultaneously. Our knowledge of the meson–meson interactions is based mainly on the reactions in which the kaon or pion pairs are produced. The production processes of the scalar mesons $f_0(975)$ and $a_0(980)$ (which both decay into the $K\overline{K}$ pairs) have been studied in many experiments [1, 2] and new experiments like those at COSY (Jülich) [3], DAΦNE (Frascati) [4, 5] and CEBAF (Newport News) are planned. Unfortunately the existing $K\overline{K}$ and $\pi\pi$ data are not sufficiently precise to construct a unique model explaining

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the nature of the poorly known scalar mesons. Therefore different theoretical approaches to this question exist (see for example refs. [6–12]).

In order to compare various models of the $K\overline{K}$ interactions we propose to calculate in future for each theoretical framework the low energy $K\overline{K}$ parameters using the effective range approximation known for example from the studies of the nucleon–nucleon interactions [13]. These parameters are crucial in understanding the nature of the K–$\overline{K}$ interactions. The importance of computing the threshold parameters has been also recently stressed by Törnqvist [14]. The masses of the $f_0(975)$ and $a_0(980)$ mesons are very close to the $K\overline{K}$ threshold. Therefore these mesons are frequently interpreted as the quasibound states of the $K\overline{K}$ pairs [15–19]. In ref. [8] the $K\overline{K}$ scalar–isoscalar scattering length has been already calculated using a separable potential formalism. Then Wycech and Green have used its value to discuss a production of the kaonic atoms [20]. More recently we have extended the calculations of the scalar–isoscalar $K\overline{K}$ and $\pi\pi$ scattering amplitudes using the relativistic approach [9]. A simple rank–one separable potential has been used to describe the $K\overline{K}$ interaction and a rank–two potential in the $\pi\pi$ channel. Choosing the rank-two potential responsible for the coupling of two channels we have obtained very good fits to the data starting from the $\pi\pi$ threshold up to 1400 MeV, thus fully covering the interesting region of the $K\overline{K}$ threshold near 1 GeV [21]. In this procedure we have been able to fix the parameters of the meson–meson interactions. As a next step we report the results of the calculations of the threshold parametrs for the $K\overline{K}$ and $\pi\pi$ interactions in the spin and isospin zero state.

We use the effective range expansion in the $\pi\pi$ and $K\overline{K}$ channels:

$$k\,cot\delta = \frac{1}{a} + \frac{1}{2}rk^2 + vk^4 + O(k^6), \tag{1}$$

where δ is the scattering phase shift, k is the relative meson momentum, a is the scattering length, r is the effective range of the interaction and the parameter v can be related to the shape of the intermeson potentials.

The low momentum expansion of the phase shift has a polynomial form:

$$\delta = \alpha k + \beta k^3 + \gamma k^5 + O(k^7). \tag{2}$$

The coefficients α, β, γ can be obtained from the low momentum expansion of the scattering amplitudes calculated in ref. [9].

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Above the $K\overline{K}$ threshold we define the complex $K\overline{K}$ phase shift $\delta = \delta_K + i\rho$, where δ_K is the $K\overline{K}$ phase shift and ρ is related to the inelasticity parameter

$$\eta = e^{-2\rho}. \tag{3}$$

In the $K\overline{K}$ channel the expansions (1) and (2) can still be valid if we make the parameters a, r, v and α, β, γ complex. From Eqs. (1) and (2) one can derive the following relations between these parameters:

$$a = \alpha, \tag{4}$$

$$r = -\frac{2}{3}\alpha - 2\frac{\beta}{\alpha^2}, \tag{5}$$

$$v = -\frac{1}{45}\alpha^3 - \frac{1}{3}\beta + \frac{\beta^2}{\alpha^3} - \frac{\gamma}{\alpha^2}. \tag{6}$$

Table 1: Low momentum parameters of the $\pi\pi$ scalar, $I = 0$ scattering

Set No	$a_\pi(m_\pi^{-1})$	$r_\pi(m_\pi^{-1})$	$v_\pi(m_\pi^{-3})$
1	0.172 ± 0.008	-8.60	3.28
2	0.174 ± 0.008	-8.51	3.25

The effective range parameters are given in tables 1 and 2 for two sets of experimental data analysed in [9]. These data sets differ qualitatively in a vicinity of the $K\overline{K}$ threshold as shown in fig. 3 of [9]. The $K\overline{K}$ phase shifts tend to decrease at threshold for the set 1 and increase for the set 2. The model [9] describes better the data set 1 than the set 2.

Table 2: Low momentum parameters of the $K\overline{K}$ scalar, $I = 0$ scattering

Set No	a_K fm	r_K fm	v_K fm^3	R_K fm	V_K fm^3
1	$-1.73 + i\,0.59$	$-0.057 + i\,0.032$	$0.016 - i\,0.0044$	0.38	-0.66
2	$-1.58 + i\,0.61$	$-0.352 + i\,0.043$	$0.028 - i\,0.0057$	0.20	-0.83

In table 2 we have introduced two additional complex parameters R_K and V_K entering into the familiar expansion valid for the real δ_K:

$$k\cot\delta_K = \frac{1}{\mathrm{Re}\,a_K} + \frac{1}{2}R_K k^2 + V_K k^4 + O(k^6). \tag{7}$$

These parameters are not independent on a_K, r_K and v_K but have been introduced for a convenience and a further discussion. Let us notice that at least four real parametrs have to be phenomenologically determined in the $K\overline{K}$ channel under the condition that one uses only two terms of the effective range expansion (1). This is in contrast to the case of the low energy proton–neutron scattering in the $^3S_1$ state (as discussed by Törnqvist in ref. [14]) since in the latter case the scattering is purely elastic.

For a full description of the two complex $\pi\pi$ and $K\overline{K}$ channels (including the $K\overline{K} \longrightarrow \pi\pi$ annihilation process) we introduce a real and symmetric matrix M related to the scattering matrix T by

$$M = T^{-1} + i\,\widehat{k} \tag{8}$$

where $\widehat{k}$ is a diagonal 2×2 matrix of the $K\overline{K}$ and $\pi\pi$ momenta in the center–off–mass system. If we label by 1 the $K\overline{K}$ channel and by 2 the $\pi\pi$ channel then the T–matrix elements read:

$$T_{11} = (2ik_1)^{-1}(\eta e^{2i\delta_1} - 1), \tag{9}$$

$$T_{22} = (2ik_2)^{-1}(\eta e^{2i\delta_2} - 1), \tag{10}$$

$$T_{12} = T_{21} = \tfrac{1}{2}(k_1 k_2)^{-1/2}(1 - \eta^2)^{1/2}e^{i(\delta_1+\delta_2)}. \tag{11}$$

At the $K\overline{K}$ threshold the M–matrix elements can be expanded as

$$M_{ij} = A_{ij} + \tfrac{1}{2}B_{ij}k_1^2 + C_{ij}k_1^4 + O(k_1^6), \tag{12}$$

where A_{ij}, B_{ij} and C_{ij} are real coefficients and k_1 is the $K\overline{K}$ momentum (i, j=1,2). Every threshold parameter in two channels introduced in eq. (1) can be related to a set of the M_{ij} expansion parameters. For example the complex $K\overline{K}$ scattering length is

$$a_K = \left(A_{11} - \frac{A_{12}^2}{A_{22} - iq}\right)^{-1}, \tag{13}$$

where $q = (m_K^2 - m_\pi^2)^{1/2}$ is the pion momentum at the $K\overline{K}$ threshold.

We use the average pion mass $m_\pi = \frac{1}{2}(m_{\pi^\pm} + m_{\pi^0}) \approx 137.27$ MeV and the average kaon mass $m_K = \frac{1}{2}(m_{K^\pm} + m_{K^0}) \approx 495.69$ MeV. The coefficients A_{ij}, B_{ij} and C_{ij} are shown in table (3) for the data set 1.

Table 3: M–matrix expansion parameters at the $K\overline{K}$ threshold

reaction channel	i	j	A_{ij} fm^{-1}	B_{ij} fm	C_{ij} fm^3
$K\overline{K}$	1	1	-0.483	-8.10×10^{-2}	1.83×10^{-2}
$\pi\pi$	2	2	0.476	-1.58×10^{-1}	1.43×10^{-3}
$K\overline{K} \longleftrightarrow \pi\pi$	1	2	0.669	-1.57×10^{-2}	5.93×10^{-3}

At first let us discuss the $\pi\pi$ threshold parameters. The $\pi\pi$ scattering length is small and positive while the $\pi\pi$ effective range is negative and much larger. The third parameter (sometimes called the shape parameter) is positive in our model. In a recent analysis of the near threshold $\pi N \longrightarrow \pi\pi N$ data D. Počanić et al. [22] have provided the $\pi\pi$ scattering length $a = (0.177 \pm 0.006) \, m_\pi^{-1}$ which is in a very good agreement with our predictions [9] (compare the second column of table 1). In the earlier analyses Lowe et al. [23] and Burkhardt and Lowe [24] have given the $\pi\pi$ scattering length values $(0.207 \pm 0.028) \, m_\pi^{-1}$ and $(0.197 \pm 0.01) \, m_\pi^{-1}$, respectively. Using the chiral perturbation theory Gasser and Leutwyler [25] have obtained a value $(0.20 \pm 0.01) \, m_\pi^{-1}$ while in a recent paper by Roberts et al. [26] the calculated values of the scattering length are $0.16 \, m_\pi^{-1}$ or $0.17 \, m_\pi^{-1}$.

The $\pi\pi$ effective range is not well determined experimentally. Belkov et al. [27] have obtained $r_\pi = (-9.6 \pm 19.1) \, m_\pi^{-1}$. Based on the analysis of the $\pi^- p \longrightarrow \pi^+ \pi^- n$ data performed by Belkov and Buniatov [28] we have derived the value of the effective range $r_\pi = -8.1 \, m_\pi^{-1}$ with an estimated error at least 65%. Within the Weinberg approach [29] the parameter $r_\pi = -8.48 \, m_\pi^{-1}$ which is very close to our values about $-8.6 \, m_\pi^{-1}$ or $-8.5 \, m_\pi^{-1}$ given in table 1 (the scattering length used in the Weinberg model was 0.157 m_π^{-1}). The effective range $(-7.4 \pm 2.5) \, m_\pi^{-1}$ can be obtained from two low energy parameters a and b predicted in ref. [25]. It is also possible to evaluate the effective range from the similar parameters fitted to the $\pi\pi$ phase shifts by Rosselet et al. [30] in the study of the K_{e4} decays ($a = 0.28 \pm 0.05$,

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$b = 0.19 - (a - 0.15)^2)$. Its value is $(-1.4 \pm 3.7) \; m_\pi^{-1}$ which is considerably different from the above cited value -8.5 fm. Another estimation based on the same data using a and b as free parameters leads to a different value $r_\pi = (0.3 \pm 6.3) \; m_\pi^{-1}$. We infer from these numbers that the existing $\pi\pi$ data are not yet substantially accurate to determine the effective range with a good precision.

The effective range expansion (1) in the $\pi\pi$ channel has a limited convergence range due to a presence of the left–hand cuts in the Mandelstam variable $s = 4(m_\pi^2 + k^2)$. In the momentum plane k there are two cuts starting at $k = \pm i m_\pi$ (see also fig. 5 of ref. [9]). These cuts lie very close to the $\pi\pi$ threshold and lead to a negative contribution to the $\pi\pi$ scattering length $(-0.18 \; m_\pi^{-1})$. The second negative contribution $(-0.24 \; m_\pi^{-1})$ comes from the singularities of the $\pi\pi$ interaction. The dominant positive contribution to a_π has its origin in a presence of the $f_0(500)$ pole in the $\pi\pi$ scattering amplitude $(+0.60 \; m_\pi^{-1})$. In the practical applications of the effective range formula the experimental data should be carefuly selected from a $\pi\pi$ momentum range very close to the threshold in order to diminish the contribution of higher terms usually neglected in the analyses. The $\pi\pi$ energy corresponding to the maximum momentum at which the convergence limit is attained in the presence of the above–mentioned cuts is as low as 390 MeV.

The $K\overline{K}$ scattering length is complex in presence of the open annihilation channel. Modulus of its real part is much larger than the $\pi\pi$ scattering length. The imaginary part is positive and gets a value about 0.6 fm. As seen in table 2 the expansion parameters r and v are rather small. This is not accidental and can be easily understood if one notices a fact that the S–matrix pole $f_0(975)$ is very close to the $K\overline{K}$ threshold. Its position in the $K\overline{K}$ momentum frame is $p_0 = (-34.7 + i\,100.3)$ MeV for the set 1 and $p_0 = (-36.1 + i\,100.2)$ MeV for the set 2. If we approximate the $K\overline{K}$ element of the S–matrix by its dominant pole contribution:

$$S_{K\overline{K}}^{pole} = \frac{-k - p_0}{k - p_0},\tag{14}$$

then the $K\overline{K}$ scattering length is $a_0 = (ip_0)^{-1}$ (see also ref. [21]) and all other parameters of the threshold expansion of $k\,cot\delta$ identically vanish since $k\,cot\delta \equiv 1/a_0$. Therefore in the single $f_0(975)$ pole approximation the parameters r_K and v_K are zero. Their smallness in the full model calculation is a reflection of the $f_0(975)$ dominance near the $K\overline{K}$ threshold. The values

a_0 are $(-1.76 + i\,0.61)$ fm for the set 1 and $(-1.74 + i\,0.63)$ fm for the set 2; they are quite close to the values a_K given in table 2 especially for the set 1 preffered by our model. The negative sign of Rea_K is characteristic for the appearence of a bound $K\overline{K}$ state $f_0(975)$. We have studied an accuracy of the pole approximation (14) in comparison with the results calculated from the complete model. For the model parameters fitted to the data set 1 both the $K\overline{K}$ phase shifts and the inelasticity are reproduced with a precision better than 2% for the $K\overline{K}$ momenta as large as 380 MeV/c (or the effective mass as high as 1250 MeV). For the set 2 the inelasticity parameter is described within 3% up to 450 MeV/c but the phase shifts are less accurately reproduced (to 11% at the threshold and up to 17% at 400 MeV/c). At the energies higher than 1250 MeV the $f_0(1400)$ resonance plays an important role and gives an additional contribution to the $f_0(975)$ term.

The $K\overline{K}$ effective range parameter R_K is relatively small in comparison with $|\,\mathrm{Re}\,a_K\,|$. The contribution of the $f_0(975)$ pole to the third parameter V_K shown in table 2 is also dominant. In this approximation both parameters R_K and V_K are given in terms of Rea_K and Ima_K. If the kaon momentum increases then the higher terms in the threshold expansion become important. The convergence radius of the expansions (2) and (7) is equal to a distance $|\,p_0\,|$ to the nearest S–matrix pole. The energy corresponding to $k = |\,p_0\,|$ is 1014 MeV which is only 23 MeV above the $K\overline{K}$ threshold. Therefore one can draw a severe limit on the experimental energy resolution needed in the determination of the $K\overline{K}$ threshold parameters. In practice one should require the energy resolution of the order of 1 MeV. The expansion (12) of the M–matrix, however, has a larger convergence radius 495.69 MeV/c limited by the kaon mass.

According to our knowledge the experimental information about the $K\overline{K}$ threshold parameters is almost nonexistent. We are aware of only one pioneer experimental determination of the $K_S^0 K_S^0$ scattering length by Wetzel et al. [31]. Although the values obtained by authors of [31] ($|\,a\,| = (1.25 \pm 0.12)$ fm, Im$a = (0.27 \pm 0.03)$ fm) are of the same order as our determinations, we think that their errors are too small. There are at least two reasons to believe that this observation is true: firstly only two experimental points are used in the analysis for the $K\overline{K}$ effective mass smaller than 1.1 GeV and secondly their parametrization of the $K\overline{K}$ phase shifts does not fulfil the general symmetry requirement: $\delta_{K\overline{K}}(-k) = \delta_{K\overline{K}}(k)$. Nevertheless these data seem to indicate a fact that the modulus of the $K\overline{K}$ scattering length is much larger than the

$\pi\pi$ scattering one.

In conclusion, we have determined the effective range parameters of the $\pi\pi$ and $K\overline{K}$ scalar–isoscalar interactions. We hope that our predictions will be confronted in future with new data clearly needed to understand the nature of the scalar mesons.

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